Estimation of a Healthcare System based on the Importance Analysis

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Abstract. Most of modern systems are complex and include heterogeneous sub-systems and components, such as software, devices, equipments, that are influenced human factor. Healthcare systems are typical example of such systems that have to perfect the care of a patient and to have high reliability. Therefore the reliability analysis of healthcare system is an important problem. There are different methods in reliability engineering for analysis and quantification of every of healthcare components. But tendency in reliability analysis needs new methods that evaluate the system as a single whole. In accordance with this tendency one aspect of reliability engineering as importance analysis is considered. The importance analysis is one of approaches in reliability engineering. In this paper the new algorithms for the calculation of importance measures are developed. These measures permit to estimate the influence of any component (sub-system) to a healthcare system functioning or failure.

1 Introduction

There is one principal characteristic for all healthcare systems. It is reliability that is defined as the probability that a system will perform its intended function during a period of running time without any failure. The system reliability is a complex characteristic that depends on the functioning of separate parts (components) of the system and depends on the combination of number of interrelated processes of every component as degradation, failure and repair, which result from the interaction of different part including not only the hardware but also the software, the human and the organizational system parts.

Application of reliability engineering methods for healthcare system investigation has been proposed more 30 years ago in paper [Taylor 1972]. E.F.Taylor declared principal items of reliability engineering in a healthcare system as reliability analysis of medical equipment and devices. Reliability quantification of equipment and devices has been principal tendency in medicine until recently. Information technology development causes application new type of healthcare systems that consist of two interdependent components as hardware and software [Cohen 2004; Dhillon 2000]. Therefore new methods for shared analysis of hardware and software part of a healthcare system have been developed. One of these methods has been presented in paper [Taleb-Bendiab et al. 2006]. But human errors problem in a healthcare system has been considered as independent problem of reliability analysis [Deeter and Rantanen 2012, Lyons et al. 2004].

Based on bibliography in reliability analysis of the healthcare domain, we can show two principal approaches of reliability engineering in medicine. The first of them is reliability estimation of medical equipment and devices that includes reliability quantification of hardware and software of the healthcare system [Dhillon 2000, Spyrou et al 2008, Taleb-Bendidb et al 2006]. The second approach agrees with examination of human errors [Bogner 1994, Deeter and Rantanen 2012, Lyons et al 2004]. However, independent evaluation of these principal parts of the healthcare system is not allowed to have detail and actual reliability analysis. In paper [Zaitseva et al 2011], new tendencies in reliability engineering are considered. According to [Zaitseva et al 2011], the reliability analysis has to base on joint evaluation of all principal parts (components).

The healthcare system typical structure consists of some principal components in the point of view of reliability analysis [Zio 2009, Dhillon 2003, Zaitseva et al 2011]. In papers [Dhillon 2000, Dhillon 2003], two of them have been defined: equipment/device and human factor. We need to note that the human factor has been considered as errors of operators of medical equipment or devices in [Dhillon 2000]. A detailed structure of the human factor and human errors for the healthcare system is presented in [Dhillon 2003]. The healthcare system structure includes three components (technical, human and organization) [Zaitseva et al 2011]. The technical component includes two types of medical devices/equipment that are based on special and standards-based technologies according to [Zio 2009, Zaitseva et al 2011]. For example, the first type is the medical decision support system, the system for integration electronic medical records or picture archiving communication systems. The second type is the special medical device and equipment that can be used for a special operation only (as magnetic resonance imaging scanners, for example). The human component of the healthcare system models medical errors. The organization component of the system joins management aspects and maintenance of the healthcare system.

The important problem in the reliability analysis of healthcare system is development of methodology that permits to investigate every system component and the system based on united approaches. This conception has been presented in papers [Zaitseva et al 2011, Zaitseva et al 2013]. We propose and develop one of possible approaches for investigation of healthcare system reliability. It is importance analysis that allows estimate the influence of every system component state change to the system performance (reliability/availability). The initial system in such analysis is declared as the *Multi-State System* (MSS). MSS is mathematical model in the reliability analysis that represents system performance behaviour as the reliability with some level: from complete failure to perfect functioning. This interpretation of system allows investigating different performance levels of system functioning, that didn't include level of functioning and fault only.

The mathematical background of the MSS importance analysis is Direct Partial Logic Derivatives [Zaitseva 2012]. The advantage of this approach is the possibility to use it for estimation of every system and its components. For example, in the papers [Zaitseva at al 2011] the application of this approach for the estimation of a healthcare system of 4 components has been considered. In this paper new algorithms for calculation of importance measures as Birnbaum importance and Fussell-Vesely importance are developed.

The paper is organized as follows. In the following Section 2, the basic conception of healthcare system reliability analysis is introduced. Then Section 3 presents mathematical model of healthcare system based on MSS and new algorithms for the calculation of the importance measures. Numerical example for the human module as part of the health care system is provided in this section too.

2 Reliability Analysis of Healthcare System

The reliability analysis of a system includes three principal steps [Zio 2009]:

- the quantification of the system model;
- the representation and modelling of the system;
- the representation, propagation and quantification of the uncertainty in the system behavior.

Consider specifics of these steps for the healthcare system in more details.

2.1 Quantification of the System

Definition of the number of performance level for the system in reliability analysis is a principal step that has influence to the development of a mathematical model too. As a rule two approaches are used for the quantification in reliability engineering.

The first of them defines only two states for the system reliability: the functioning and failure. The mathematical model for the representation of this quantification is named as a Binary-State System (BSS). This approach is well known and widely used in reliability engineering. The system failure can be investigated in detail based on this quantification. However, the analysis of other performance level before the system failure has some difficulties for BSS. In this case, the quantification of the system reliability to some performance levels is used. The mathematical model with some performance level is named Multi-State System (MSS).

MSS reliability analysis is a more flexible approach to evaluating system reliability, as it can be used when both the system and its components may experience more than two states, including, for example, completely failed, partially failed, partially functioning and perfect functioning [Lisnianski and Levitin 2003, Natvig 2011]. However, a mathematical approach to analysing such a system is complex.

2.2 Modelling of the System

The next step after the definition of quantification is the mathematical model development. There are some types of the BSS representation as the mathematical model. These representations (mathematical models) correlate with the mathematical methods for the calculation of the system reliability indices and measures. For example, the description of the system by the Universal Generating Function is used in the system reliability optimization [Li and Zio 2012]. One other of these representations is the structure function. This function allows the mathematical description of a system with any complexity [Lisnianski and Levitin 2003, Zaitseva 2012].

The *structure function* defines the system state (system reliability/performance level) depending on the system components states. According to the definition of the structure function the system reliability in the stationary state is defined as [Zaitseva 2012]:

$$\phi(x_1,\ldots,x_n) = \phi(\boldsymbol{x}) : \{0,\ldots,m_1-1\}\times\ldots\times\{0,\ldots,m_n-1\}\rightarrow\{0,\ldots,M-1\} \quad (1)$$

In (1) the x_i is the state of the i-th system component that can be defined form 0 (the component failure) to m_i-1 (the perfect component performance level): $x_i \in \{0, \ldots, m_i-1\}$, and the system reliability has M level from 0 (as the failure) to M-1 (as the perfect functioning). Note, the system component has different number of states: $m_i \neq m_k$, if $i \neq k$ ($i, k \in \{1, \ldots, n\}$). The number of the system component is declared as n.

The structure function definition (1) is definition for a MSS, where system and its component have some (more than two) performance levels. The structure function for a BSS is defined as the Boolean function [Zaitseva and Levashenko 2013]:

$$\phi(x_1,\ldots,x_n) = \phi(x) : \{0,1\}^n \rightarrow \{0,1\} \quad (2)$$

We will consider a coherent system in this paper. Such system has two principal assumptions for the structure function [Lisnianski and Levitin 2003]: (a) the structure function (1) and (2) is monotone, and (b) the system component state decrease does not improve the system reliability.

Every system component is characterized by probability of the component state:

$$p_{i,s_i} = \Pr\{x_i = s_i\},\ s_i \in \{0, \ldots, m_i\text{-}1\} \tag{3}$$

Note, the structure function (1) and (2) present the investigated object (system) in the stationary state. The system behavior and correlation of changes of components states and system reliability can be defined by mathematical tools of Logical Differential Calculus, in particular the Direct Partial Logic Derivative. The Direct Partial Logic Derivative with respect to variable x_i for the structure function (1) permits to analyse the system reliability change from j to $\bar{j}$ when the i-th component state changes from a to $\bar{a}$ [Zaitseva 2012, Zaitseva and Levashenko 2013]:

$$\partial\phi(j \to \bar{j})/\partial x_i(a \to \bar{a}) = \\ = \begin{cases} 1, & if\ \phi(a_i, \boldsymbol{x}) = j\ \&\ \phi(\bar{a}_i, \boldsymbol{x}) = \bar{j} \\ 0, & other \end{cases} \tag{4}$$

where $\phi(a_i, \boldsymbol{x}) = \phi(x_1, \ldots, x_{i-1}, a, x_{i+1}, \ldots, x_n)$; $\phi(\bar{a}, \boldsymbol{x}) = \phi(x_1, \ldots, x_{i-1}, \bar{a}, x_{i+1}, \ldots, x_n)$; $a, j \in \{0, 1\}$ and $\bar{a} = 1\text{-}a$, $\bar{j} = 1\text{-}j$.

Let us consider the system failure in the Direct Partial Logic Derivative terminology. The system failure is represented as a change of the structure function value $\phi(\boldsymbol{x})$ from state 1 into 0. This change can be caused by the i-th variable change from 1 to 0 if we consider a coherent system. Therefore the Direct Partial Logic Derivative for the system failure analysis is defined by the equation

$$\partial\phi(0 \to 1)/\partial x_i(a \to a-1) = \\ = \begin{cases} 1, & if\ \phi(a_i, \boldsymbol{x}) = 1\ and\ \phi((a-1)_i, \boldsymbol{x}) = 0 \\ 0, & other \end{cases}. \tag{5}$$

This derivative for the system with two performance levels (BSS) is defined as:

$$\partial\phi(1 \to 0)/\partial x_i(1 \to 0) = \\ = \begin{cases} 1, & if\ \phi(1_i, \boldsymbol{x}) = 1\ and\ \phi(0_i, \boldsymbol{x}) = 0 \\ 0, & other \end{cases}. \tag{6}$$

The Direct Partial Logic Derivative (5) allows investigating boundary states of this system for which reduction of the one component x_i causes the system failure. The Direct Partial Logic Derivative (6) describes the situation for which the system fails depending on one system component failure. Therefore, these derivatives allow calculating the system boundary states for the system reliability that are agree with vector state $\boldsymbol{x} = (x_1\ x_2\ \dots\ x_n)$ and the change of one of component state causes the change the system performance level (system reliability).

There one more type of the Direct Partial logic Derivative that allows investigating the influence of all changes of the component to the system reliability. It is union of all derivatives $\partial\phi(j \to \tilde{j})/\partial x_i(a \to (a+1))$ or $\partial\phi(j \to \tilde{j})/\partial x_i(a \to (a-1))$. These unions of derivatives identify all situations, in which the improvement of the fixed component state results into the transition of the system from state less than j to state greater than or equal to j:

$$\frac{\partial\phi(\uparrow j \uparrow)}{\partial x_i(s \to s+1)} = \bigcup_{w=0}^{j-1}\left(\bigcup_{v=j}^{M-1} \frac{\partial\phi(w \to v)}{\partial x_i(s \to s+1)}\right) \tag{7}$$

for $s = 0, \dots, m_i$-2 and $j = 1, \dots, M$-1, $w < j$, $v \geq j$.

The Direct Partial Logic Derivative union (7) can be defined as:

$$\frac{\partial\phi(\uparrow j \uparrow)}{\partial x_i(s \to s+1)} = \begin{cases} 1 & \text{if } \phi(s_i, \boldsymbol{x}) \leq j \text{ and } \phi((s+1)_i, \boldsymbol{x}) > j \\ 0 & other \end{cases} \tag{8}$$

for $s = 0, \dots, m$-2 and $j = 1, \dots, m$-1

The dimension of the Direct Partial Logic Derivative with respect to the variable x_i is defined as $m^{n\text{-}1}$ [Tapia et al 1991]. Therefore the Direct Partial Logic Derivative union (7) and (8) for the i-th variable has dimension $m^{n\text{-}1}$ too. But the structure function investigation supposes the analysis of the all possible system states. Therefore the Direct Partial Logic Derivative union (7) and (8) has to be transformed for dimension m^n as:

$$\frac{\partial\phi(\uparrow j \uparrow)}{\partial x_i(s \to s+1)} = \\ = \begin{cases} 1 & \text{if } x_i = s \text{ and } \phi(s_i, \boldsymbol{x}) \leq j \text{ and } \phi((s-1)_i, \boldsymbol{x}) > j \\ 0 & \text{if } x_i = s \text{ and } \left(\phi(s_i, \boldsymbol{x}) > j \text{ or } \phi((s-1)_i, \boldsymbol{x}) \leq j\right), \\ * & \text{if } x_i \neq s \end{cases} \tag{9}$$

The merge (union) of unions (9) of Direct Partial Logic Derivatives has to be computed to identify all situations, in which any improvement of a given component causes the transition of the system from state less than j to state greater than or equal to j:

$$\frac{\partial\phi(\uparrow j \uparrow)}{\partial x_i} = \bigcup_{s=0}^{m-2} \frac{\partial\phi(\uparrow j \uparrow)}{\partial x_i(s \to s+1)} \quad \text{for } j = 1,2,\ldots,m-1. \tag{10}$$

The intersection of two merges (10) of two different variables (components) is defined in Table I. This intersection identifies state vectors, in which the change of states of both components (if component state can be changed) results in the change of the system state from value less than j to value greater than or equal to j. In Table I, symbol "1" means that at least state of one component from components i_1 and i_2 can be changed and all those changes result in the required change of the system state (from state less than j to state greater than or equal to j). Symbol "0" identifies those state vectors, in which at least one component change does not cause the required change of the system state. Finally, symbol "*" correlates with those situations, when neither component i_1 nor component i_2 can be changed from state s to state s +1, because both components are in state m -1.

Table 1. The intersection of two merges of unions of Direct Partial Logic Derivatives

		$\frac{\partial\phi(\downarrow j \downarrow)}{\partial x_{i_2}}$		
		*****	**0**	**1**
$\frac{\partial\phi(\downarrow j \downarrow)}{\partial x_{i_1}}$	*	*	0	1
	0	0	0	0
	1	1	0	1

The intersection of two merges of extended unions of Direct Partial Logic Derivatives identifies state vectors, in which the improvement of both components (if component can be repaired, i.e. if component is not in state M -1) results in the improvement of the system from state less than j to state greater than or equal to j.

The merge (10) of extended unions of Direct Partial Logic Derivatives has to be computed for all components of the system and then their "intersection" allows identifying *Minimal Cut Sets* (MCVs) for system state j. The MCS is one more type of boundary states of the system for which the component state improve cause the system reliability change. The MCS is one of basic conceptions in reliability engineering that is used for the calculation a lot of reliability measures and indices.

Let us consider the conception of the minimal cut set (MCS). MCS methods have been proposed for BSS reliability analysis in the first. According to [Shooman 1968, Al-Muhaini and Heydt 2012] a cut set is defined as a set of components of a system whose simultaneous failure leads into the failure of the system (if the system has been operational). A cut set is minimal, if no component can be removed from it without losing its status as a cut set.

In the terms of the structure function, a (minimal) cut set can be interpreted by a special state vector, which is known as a Minimal Cut Vector (MCV). According to the definition of cut set, the system state for a state vector covered by a cut set is zero. Therefore state vector x is a cut vector if $\phi(x) = 0$.

So, if all components of a cut set are failed and components out of the cut set are functioning then the system is failed. A state vector, which coincides with a MCS, is known as a MCV. Using the convention that $y > x$, where x and y are two states vector, for which $y_i \geq x_i$ (for $i = 1,2,\ldots,n$) and there exists at least one i such that $y_i > x_i$, we say that a cut vector x is minimal if $\phi(y) = 1$ for any $y > x$.

There is one-to-one correspondence between MCSs and MCVs. However, the terms MCS and MCV are slightly different. A MCS is a minimal set of components, whose simultaneous failure causes the system failure, while MCV represents situation in which the repair of any failed component results into the repair of the whole system.

The definition of MCS has been generalized for MSSs in paper [Butler 1979]. The development of this conception for MCVs of a MSS has been proposed in [Lisnianski and Levitin 2003, Soh and Rai 2005, Yeh 2008].

The generalization of MCV definition for a MSS takes into account that components of a MSS have more than two states. This extension is based on the assumption that MCV is defined for every relevant system state, i.e. for states $\{1, 2,\ldots, m -1\}$ The definition of a MCV of a MSS is proposed in [Lisnianski and Levitin 2003] as follows: a state vector x is a cut vector for demand state j of the system if $\phi(x) < j$. A cut vector x is minimal if $\phi(y) \geq j$ for any $y > x$.

The meaning of MCVs for a MSS is similar as in the case of a BSS, i.e. MCVs for system state j identify those situations in which the repair of any damaged component causes the improvement of the system at least to state j.

2.3 Mathematical Method

The boundary states are one of basic conception in the reliability analysis and used in different mathematical methods for the computation of reliability indices and measures. For example boundary state is principal item in the investigation based on Fault Tree [Contini and Matuzas 2011, Farcasiu and Prisecaru 2014, Merle et al 2011]. These states are considered and analysed in the method of *Failure Models and Effect Analysis* (FMEA) [Farcasiu and Prisecaru 2014, Seyed-Hosseini et al 2006]. The boundary states are used in Importance analysis for the computation of the *Importance Measures* (IM) [Vaurio 2010, Zaitseva2012]. Importance analysis allows examining different aspects of reliability changes and the uncertainty in the system. IM quantifies the criticality of a particular component within the system. They have been widely used as tools for identifying system weaknesses, and to prioritise reliability improvement activities.

The most used IMs as *Structural Importance* (SI), *Birnbaum importance* (BI), *Fussell-Vesely importance* (FVI) are shown in Table 2 [Lisnianski and Levitin 2003, Zaitseva 2012].

Table 2. Importance measures

Short name	Description
SI	SI concentrates on the topological structure of the system and determines the proportion of working states of the system in which the working of the *i*-th component makes the difference between system failure and working state
BI	BI of a given component is defined as the probability that such a component is critical to MSS functioning and represents loss in MSS when the *i*-th component fails.
FVI	FVI quantifies the maximum decrement in MSS reliability caused by the *i*-th system component state deterioration and if $s = 0$, the measure allows estimating system performance level decrease for full unreliability of the *i*-th system component.

The Direct Partial Logic Derivative is one of possible approaches for calculation of IMs [Zaitseva 2012]. According to [Zaitseva 2012], the Direct Partial Logic Derivative has been used for calculation of SI and BI. The FVI definition is based on the MCS that is calculated based on the Direct Partial Logic Derivative (10) too.

In this a paper we propose unify method for calculation of the IMs based on the Direct Partial Logic Derivative (5), (9), (10). These measures can be used for the estimation of the system component that has maximal influence to the system functioning (system availability). In this paper we propose to apply such analysis for the investigation of the healthcare system.

3 Importance Analyses

The importance analysis is part in reliability analysis that allows investigating the structural and topological aspects of the system in point of view of the reliability. The IMs permit to discover the system component with maximal/minimal influence of the system reliability/availability. This information is useful for system design and maintains.

3.1 Structural Importance

SI is one of the simplest measures of the component importance and this measure focuses on the topological aspects of the system. According to the definition in paper [Armstrong 1997], this measure determines the proportion of working states of the system in which the working of the i-th component makes the difference between system failure and its working:

$$IS_i^{s,j} = \frac{\rho_i^{s,j}}{\rho_s} \quad (11)$$

where $\rho_i^{s,j}$ is a number of system states when the change component state from s to s-1 results in the system reliability change from j to j-1; ρ_s is number of the states for which $\phi(s_i, x) = j$ and is calculated by the structure function.

The number $\rho_i^{s,j}$ can be computed as the number of nonzero values of the Direct Partial Logic Derivative (5).

For example, consider the human module of the health care system from the paper [Lyons et al 2004] that is represented by the Fault Decision Tree (Fig.1). Transform this module to the MSS (Fig.2) with the structure function defined in the Table 3.

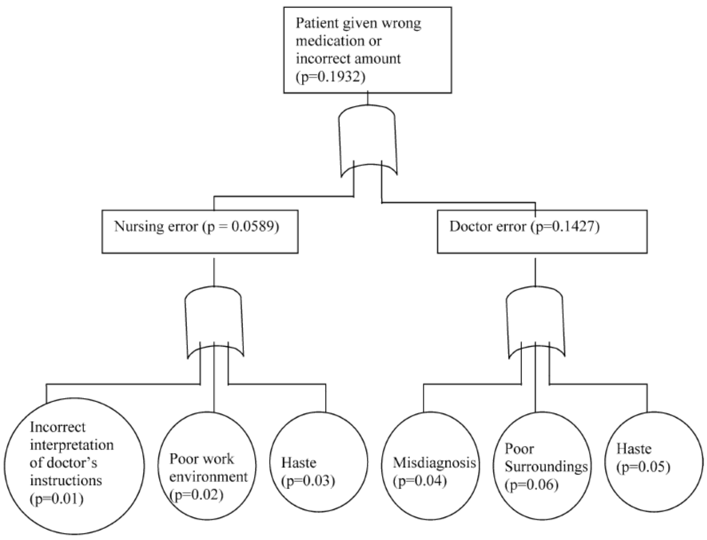
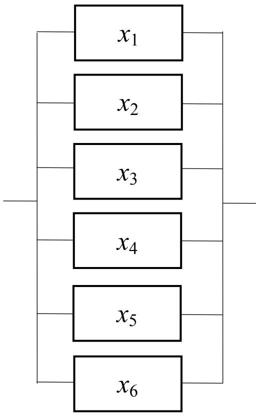


Fig. 1 The human module (from the paper [Lyons et al 2004])



	$p_{0,i}$	$p_{1,i}$
Incorrect interpretation of doctor's instructions, x_1	0.01	0.99
Poor work environment, x_2	0.02	0.98
Haste, x_3	0.03	0.97
Misdiagnosis, x_4	0.04	0.96
Poor Surroundings, x_5	0.06	0.94
Haste, x_6	0.05	0.95

Fig. 2 The human module adapted to the MSS from the paper [Lyons et al 2004]

Table 3. The structure function of the MSS for the human module in Fig.2

	$x_1\ x_2\ x_3$							
$x_4\ x_5\ x_6$	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
0 0 0	0	1	1	1	1	1	1	2
0 0 1	1	1	1	1	1	1	1	2
0 1 0	1	1	1	1	1	1	1	2
0 1 1	1	2	2	2	2	2	2	3
1 0 0	1	1	1	1	1	1	2	2
1 0 1	1	2	2	2	2	2	2	3
1 1 0	1	2	2	2	2	2	2	3
1 1 1	1	2	2	3	2	3	3	3

Values of SI (11) and intermediate values of $\rho_i^{s,j}$ are shown in Table 4. Therefore the SI values in the Table 4 permit to estimate the influence of the every human errors to the correct medical decision. The fatal influence ($IS_i^{1,1}$) to the medical mistake has doctor's errors (Fig. 1) because the values of SI are maximal for x_3 ($IS_3^{1,1}$=0.1111), x_4 ($IS_4^{1,1}$=0.1000) and x_5 ($IS_5^{1,1}$=0.1000). These errors have principal influence for the some un-corrections in the decision that are defined a performance level 1: $IS_3^{1,2}$=0.7647, $IS_4^{1,2}$=0.6875 and $IS_5^{1,2}$=0.6875. The insignificant disadvantages (that agree with performance level 2) are caused by nurses and doctors errors equally.

Table 4. Structural importance for the human module in Fig.2

i	$\rho_{s,j}$			$\rho_i^{s,j}$			IS_i		
	$\rho_{1,1}$	$\rho_{1,2}$	$\rho_{1,3}$	$\rho_i^{1,1}$	$\rho_i^{1,2}$	$\rho_i^{1,3}$	$IS_i^{1,1}$	$IS_i^{1,2}$	$IS_i^{1,3}$
1	11	15	6	1	9	5	0.0909	0.6000	0.8333
2	11	15	6	1	9	5	0.0909	0.6000	0.8333
3	12	14	6	1	7	5	0.0833	0.5000	0.8333
4	9	17	6	1	13	5	0.1111	0.7647	0.8333
5	10	16	6	1	11	5	0.1000	0.6875	0.8333
6	10	16	6	1	11	5	0.1000	0.6875	0.8333

3.2 Birnbaum Importance

Note the SI (11) investigates correlation and influence of system component in point of view of the reliability/availability investigation. But this measure doesn't take into account the probability of state of system components. These disadvantage is eliminated in the calculation other importance measure as BI. The BI of a given component is defined as the probability that system is sensitive to inoperative of the *i*-th system component [Lisnianski and Levitin 2003]. Let us consider the Direct Partial Logical Derivatives for calculation of BI. In paper [Zaitseva 2012], BI has been defined as

$$IB_i^{s,j} = \Pr\left(\partial\phi(j \to j-1)/\partial x_i(s \to s-1)\right) \tag{12}$$

The equation (12) supposes the BI calculation as the probabilities of non-zero values of Direct Partial Logic Derivative $\partial\phi(j \to j–1)/\partial x_i(s \to s–1)$.

For example, let us consider the system shown in Fig.2. Probabilities of the system element reliability and unreliability are shown in this figure too. The Direct Partial Logic Derivatives for the first variable $\partial\phi(j \to j–1)/\partial x_1(1 \to 0)$ are in Table 5.

Table 6. Direct Partial Logical Derivatives $\partial\phi(j \to j–1)/\partial x_1(1\to0)$ for the human module in Fig.2

$x_2\,x_3\,x_4\,x_5\,x_6$	$\partial\phi(1\to0)/\partial x_1(1\to0)$	$\partial\phi(2\to1)/\partial x_1(1\to0)$	$\partial\phi(3\to2)/\partial x_1(1\to0)$
0 0 0 0 0	1	0	0
0 0 0 0 1	0	0	0
0 0 0 1 0	0	0	0
0 0 0 1 1	0	1	0
0 0 1 0 0	0	0	0
0 0 1 0 1	0	1	0
0 0 1 1 0	0	1	0
0 0 1 1 1	0	1	0
0 1 0 0 0	0	0	0
0 1 0 0 1	0	0	0

0 1 0 1 0	0	0	0
0 1 0 1 1	0	0	0
0 1 1 0 0	0	0	0
0 1 1 0 1	0	0	0
0 1 1 1 0	0	0	0
0 1 1 1 1	0	0	1
1 0 0 0 0	0	0	0
1 0 0 0 1	0	0	0
1 0 0 1 0	0	0	0
1 0 0 1 1	0	0	0
1 0 1 0 0	0	1	0
1 0 1 0 1	0	0	0
1 0 1 1 0	0	0	0
1 0 1 1 1	0	0	1
1 1 0 0 0	0	1	0
1 1 0 0 1	0	1	0
1 1 0 1 0	0	1	0
1 1 0 1 1	0	0	1
1 1 1 0 0	0	1	0
1 1 1 0 1	0	0	1
1 1 1 1 0	0	0	1
1 1 1 1 1	0	0	1

According to data in Table 5 and equation (12) the BI measures for the first component are calculated as:

$$IB_1^{1,1} = \Pr\{\partial\phi(1\to 0)/\partial x_1(1\to 0)=1\} = p_{0,2}p_{0,3}p_{0,4}p_{0,5}p_{0,6} = 0.000000072$$

$$IB_1^{1,2} = \Pr\{\partial\phi(2\to 1)/\partial x_1(1\to 0)=1\} =$$
$$= p_{0,2}p_{0,3}p_{0,4}p_{1,5}p_{1,6} + p_{0,2}p_{0,3}p_{1,4}p_{0,5}p_{1,6} + p_{0,2}p_{0,3}p_{1,4}p_{1,5}p_{0,6} + p_{0,2}p_{0,3}p_{1,4}p_{1,5}p_{1,6} +$$
$$+ p_{1,2}p_{0,3}p_{1,4}p_{0,5}p_{0,6} + p_{1,2}p_{1,3}p_{0,4}p_{0,5}p_{0,6} + p_{1,2}p_{1,3}p_{0,4}p_{0,5}p_{1,6} + p_{1,2}p_{1,3}p_{0,4}p_{1,5}p_{0,6} +$$
$$+ p_{1,2}p_{1,3}p_{1,4}p_{0,5}p_{0,6} = 0.041442$$

$$IB_1^{1,1} = \Pr\{\partial\phi(3\to 2)/\partial x_1(1\to 0)=1\} =$$
$$p_{0,2}p_{1,3}p_{1,4}p_{1,5}p_{1,6} + p_{1,2}p_{0,3}p_{1,4}p_{1,5}p_{1,6} + p_{1,2}p_{1,3}p_{0,4}p_{1,5}p_{1,6} + p_{1,2}p_{1,3}p_{1,4}p_{0,5}p_{1,6} +$$
$$+ p_{1,2}p_{1,3}p_{1,4}p_{1,5}p_{0,6} + p_{1,2}p_{1,3}p_{1,4}p_{1,5}p_{1,6} = 0.985629$$

The measures BI for all system components are in Table 6. These measures are calculated according to (12) and based on the Direct Partial Logic Derivatives for the structure function of the system (the human module) in the figure 2.

Table 6. BI for the human module in Fig.2

i	$IB_i^{1,1}$	$IB_i^{1,2}$	$IB_i^{1,3}$
1	0,000000072	0,007487	0,170699

2	0,000000036	0,007259	0,163955
3	0,000000024	0,004351	0,156896
4	0,000000018	0,006213	0,149505
5	0,000000012	0,003667	0,133664
6	0,000000014	0,003931	0,141767

Therefore the fatal result caused by the doctor's or nurse's errors has minimal probability. In this situation the "*Incorrect interpretation of doctor's instructions*" has maximal influence to the fatal result according to input data (Fig.1. and Fig.2). This error according to the structure function (Table 1) and probabilities of the module components (Fig.1) has principal influence for the other performance level too. The Bi is more informative measure because considers the structure function and the probabilities of the components states.

3.2 Fussell-Vesely Importance

There is one more measure that allows investigating influence of the component state change to the system performance level (reliability/availbility). It is FVI that represents the contribution of each component to the system failure probability and for BSS is calculated by the next equation [Meng 2009]:

$$IFV_i = \frac{F_{\text{min cut}}(x_i)}{Q} \tag{13}$$

where $F_{\text{min cut}}(x_i)$ is the system minimal cut that includes the i-th system component; Q is the function of the system unreliability [Meng 2009]:

$$Q = \Pr\{\phi(\boldsymbol{x})=0\}. \tag{14}$$

Therefore, for calculation of this measure is based on the minimal cut set, that have been considered above in section 2.

The FVI measure for the MSS is defined by the conception of the MCVs that are defined and computed according to (10). FVI for MSS is defined as [Zaitseva 2012]:

$$IFV_i^j = \frac{\Pr\{MCV_i^j\}}{1-R(j)} \tag{15}$$

where MCV_i^j is the system MCVs for system performance level j that include the i-th system component; $R(j)$ is the probability of the system performance level j [Lisnianski and Levitin 2003]:

$$R(j) = \Pr\{\phi(\boldsymbol{x}) \geq j\}; \quad U(j) = 1 - R(j) = \Pr\{\phi(\boldsymbol{x}) < j\}, \tag{16}$$

MCV_i^j are system MCVs for system performance level j that include the i-th system component and $\Pr\{MCV_i^j\}$ is the probability that system components are in such states that their improvement results system repair to performance level j or greater, i.e.:

$$\Pr\{MCV_i^j\} = \Pr\{\boldsymbol{x} \leq MCV_{i,1}^j \text{ OR } \boldsymbol{x} \leq MCV_{i,2}^j \text{ OR} \ldots \text{OR } \boldsymbol{x} \leq MCV_{i,l_i^j}^j\}, \quad (17)$$

where l_i^j is the number of MCVs for system performance level j that include component i and $MCV_{i,t}^j$ is the t-th MCV from set MCV_i^j.

In the human module in Fig. 2, the reliability of every component, i.e. probability that system component is in state 1, is high and therefore, expression (17) can be estimates as follows [Zio 2007]:

$$\Pr\{MCV_i^j\} \approx \sum_{t=1}^{l_i^j} \Pr\{\boldsymbol{x} \leq MCV_{i,t}^j\}, \quad (18)$$

and therefore the FVI measure for the human module can be computed as follows:

$$IFV_i^j \approx \frac{1}{1-R(j)} \sum_{t=1}^{l_i^j} \Pr\{\boldsymbol{x} \leq MCV_{i,t}^j\}. \quad (19)$$

Now, compute the FVI measures for individual components of the human module in Fig. 2. MCVs of the human module have to be computed firstly. In section 2, there has been mentioned that Direct Partial Logic Derivatives can be used for this task.

Assume that we want to compute MCVs for performance level 2 of the human module in Fig. 2. Firstly, the merge (10) of Direct Partial Logic Derivatives unions (9) and then their intersection have to be calculated. In Table 7, the merge of Direct Partial Logic Derivatives according to components 1, 2 and 3 (the white columns) and their intersection (the gray column) for state 2 of the human module are computed. This intersection identifies situations in which the repair of any failed component from set of components 1, 2 and 3 causes the transition of the system from performance level less than 2 to the level 2 or greater. However, MCVs for this performance levels are defined as state vectors in which the repair of any failed system component results this type of system improvement. Therefore, the merge of Direct Partial Logic Derivatives according to other components, i.e. 4, 5 and 6 and their intersection has to be calculated too (Table 8).

Table 7. The merge (10) of Direct Partial Logic Derivatives unions (9) for state 2 of the human module in Fig. 2 according to variables x_1, x_2 and x_3 and their intersection

$x_1\ x_2\ x_3$

$x_4\,x_5\,x_6$	0 0 0		0 0 1		0 1 0		0 1 1		1 0 0		1 0 1		1 1 0		1 1 1	
0 0 0	0 0 0	0	0 0 *	0	0 * 0	0	1 * *	1	* 0 0	0	* 1 *	1	* * 1	1	* * *	*
0 0 1	0 0 0	0	0 0 *	0	0 * 0	0	1 * *	1	* 0 0	0	* 1 *	1	* * 1	1	* * *	*
0 1 0	0 0 0	0	0 0 *	0	0 * 0	0	1 * *	1	* 0 0	0	* 1 *	1	* * 1	1	* * *	*
0 1 1	1 1 1	1	0 0 *	0	0 * 0	0	0 * *	0	* 0 0	0	* 0 *	0	* * 0	0	* * *	*
1 0 0	0 0 0	0	0 0 *	0	1 * 0	0	1 * *	1	* 1 0	0	* 1 *	1	* * 0	0	* * *	*
1 0 1	1 1 1	1	0 0 *	0	0 * 0	0	0 * *	0	* 0 0	0	* 0 *	0	* * 0	0	* * *	*
1 1 0	1 1 1	1	0 0 *	0	0 * 0	0	0 * *	0	* 0 0	0	* 0 *	0	* * 0	0	* * *	*
1 1 1	1 1 1	1	0 0 *	0	0 * 0	0	0 * *	0	* 0 0	0	* 0 *	0	* * 0	0	* * *	*

Table 8. The merge (10) of Direct Partial Logic Derivatives unions (9) for state 2 of the human module in Fig. 2 according to variables x_4, x_5 and x_6 and their intersection

	$x_1\,x_2\,x_3$															
$x_4\,x_5\,x_6$	0 0 0		0 0 1		0 1 0		0 1 1		1 0 0		1 0 1		1 1 0		1 1 1	
0 0 0	0 0 0	0	0 0 0	0	0 0 0	0	0 0 0	0	0 0 0	0	0 0 0	0	1 0 0	0	0 0 0	0
0 0 1	0 0 *	0	1 1 *	1	1 1 *	1	1 1 *	1	1 1 *	1	1 1 *	1	1 1 *	1	0 0 *	0
0 1 0	0 * 0	0	1 * 1	1	1 * 1	1	1 * 1	1	1 * 1	1	1 * 1	1	1 * 1	1	0 * 0	0
0 1 1	0 * *	0	0 * *	0	0 * *	0	0 * *	0	0 * *	0	0 * *	0	0 * *	0	0 * *	0
1 0 0	* 0 0	0	* 1 1	1	* 1 1	1	* 1 1	1	* 1 1	1	* 1 1	1	* 0 0	0	* 0 0	0
1 0 1	* 0 *	0	* 0 *	0	* 0 *	0	* 0 *	0	* 0 *	0	* 0 *	0	* 0 *	0	* 0 *	0
1 1 0	* * 0	0	* * 0	0	* * 0	0	* * 0	0	* * 0	0	* * 0	0	* * 0	0	* * 0	0
1 1 1	* * *	*	* * *	*	* * *	*	* * *	*	* * *	*	* * *	*	* * *	*	* * *	*

In Table 9, there is computed the intersection (the gray column) of intersections (the white columns) from Table 7 and Table 8. This intersection identifies all situations in which the repair of any failed component coincide with the transition of the human module from performance level less than 2 to the level 2 or higher, i.e. it reveals MCVs of the human module for performance level 2.

The previous process can be performed for other performance levels of the system in Fig. 2 to find MCVs for every relevant performance level of the system, i.e. for levels 1, 2 and 3.

Table 9. The intersections from Table 7 and Table 8 and their intersection

	$x_1\,x_2\,x_3$															
$x_4\,x_5\,x_6$	0 0 0		0 0 1		0 1 0		0 1 1		1 0 0		1 0 1		1 1 0		1 1 1	
0 0 0	0 0	0	0 0	0	0 0	0	1 0	0	0 0	0	1 0	0	1 0	0	* 0	0
0 0 1	0 0	0	0 1	0	0 1	0	1 1	**1**	0 1	0	1 1	**1**	1 1	**1**	* 0	0
0 1 0	0 0	0	0 1	0	0 1	0	1 1	**1**	0 1	0	1 1	**1**	1 1	**1**	* 0	0
0 1 1	1 0	0	0 0	0	0 0	0	0 0	0	0 0	0	0 0	0	0 0	0	* 0	0
1 0 0	0 0	0	0 1	0	0 1	0	1 1	**1**	0 1	0	1 1	**1**	0 0	0	* 0	0

1 0 1	1 0	0	0 0	0	0 0	0	0 0	0	0 0	0	0 0	0	0 0	0	* 0	0
1 1 0	1 0	0	0 0	0	0 0	0	0 0	0	0 0	0	0 0	0	0 0	0	* 0	0
1 1 1	1 *	**1**	0 *	0	0 *	0	0 *	0	0 *	0	0 *	0	0 *	0	* *	*

When a MCV contains value 0 at position *i*, then it is a MCV for the *i*-th component. Using this rule, MCVs for given performance level of the human module can be split into six groups (one group for one component) and the *i*-th group contains only those MCVs that have value 0 at position *i*, i.e. the *i*-th group represents MCVs for the *i*-th component (Table 10).

Table 10. MCVs for components and states of the human module in Fig. 2

System state	Component					
	1	2	3	4	5	6
1	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)
2	(0,0,0,1,1,1) (0,1,1,0,0,1) (0,1,1,0,1,0) (0,1,1,1,0,0)	(0,0,0,1,1,1) (1,0,1,0,0,1) (1,0,1,0,1,0) (1,0,1,1,0,0)	(0,0,0,1,1,1) (1,1,0,0,0,1) (1,1,0,0,1,0)	(0,1,1,0,0,1) (0,1,1,0,1,0) (1,0,1,0,0,1) (1,0,1,0,1,0) (1,1,0,0,0,1) (1,1,0,0,1,0)	(0,1,1,0,0,1) (0,1,1,1,0,0) (1,0,1,0,0,1) (1,0,1,1,0,0) (1,1,0,0,0,1)	(0,1,1,0,1,0) (0,1,1,1,0,0) (1,0,1,0,1,0) (1,0,1,1,0,0) (1,1,0,0,1,0)
3	(0,0,1,1,1,1) (0,1,0,1,1,1) (0,1,1,0,1,1) (0,1,1,1,0,1) (0,1,1,1,1,0)	(0,0,1,1,1,1) (1,0,0,1,1,1) (1,0,1,0,1,1) (1,0,1,1,0,1) (1,0,1,1,1,0)	(0,1,0,1,1,1) (1,0,0,1,1,1) (1,1,0,0,1,1) (1,1,0,1,0,1) (1,1,0,1,1,0)	(0,1,1,0,1,1) (1,0,1,0,1,1) (1,1,0,0,1,1) (1,1,1,0,0,1) (1,1,1,0,1,0)	(0,1,1,1,0,1) (1,0,1,1,0,1) (1,1,0,1,0,1) (1,1,1,0,0,1) (1,1,1,1,0,0)	(0,1,1,1,1,0) (1,0,1,1,1,0) (1,1,0,1,1,0) (1,1,1,0,1,0) (1,1,1,1,0,0)

In the next step, the probabilities $\Pr\{MCV_i^j\}$ for the *i*-th component and *j*-th system performance level can be computed using (18). These probabilities are calculated in Table 11 and they represent the contribution of the *i*-th component failure to the system unreliability (16).

Finally, when the FVI of individual components of the human module have to be calculated, the unreliabilities for individual system performance levels have to be computed (Table 12).

Table 11. Contributions of components of the human module in Fig. 2 to the module unreliability

Component (*i*)	$\Pr\{MCV_i^1\}$	$\Pr\{MCV_i^2\}$	$\Pr\{MCV_i^3\}$
1	0.00000000072	0.000080	0.0020
2	0.00000000072	0.000154	0.0038
3	0.00000000072	0.000138	0.0054
4	0.00000000072	0.000264	0.0068
5	0.00000000072	0.000234	0.0090
6	0.00000000072	0.000210	0.0080

Table 12. Reliabilities and unreliabilities of states of the human module in Fig. 2

System state (j)	$R(j)$	$U(j) = 1 - R(j)$
1	0.99999999928	0.00000000072
2	0.99965610128	0.00034389872
3	0.98392198200	0.01607801800

Using data from Table 11 and Table 12, the FVI for individual components and system performance levels of the human module in Fig. 2 are computed in Table 13. According to this table, the probability that the failure of any component contribute to the total failure of the human module, i.e. IFV_i^1 is same for all components. This is caused by the fact that there is only one MCV for system performance level 1 and it is shared by all system components.

The values of IFV_i^2 describe the probabilities that the failure of the i-th component contributes to human module unreliability (16) with respect to performance level 2 of the system. According to these values, the "*Misdiagnosis*" has the most and the "*Incorrect interpretation of doctor's instructions*" has the least contribution.

In the last column of Table 13, there are values of the FVI for performance level 3 and they indicate that the "*Poor Surroundings*" has the most contribution to the system unreliability (16) defined with regard to performance level 3 of the human module and the "*Incorrect interpretation of doctor's instructions*" has the least.

Table 13. The FVIs for the human module in Fig. 2

Component (i)	IFV_i^1	IFV_i^2	IFV_i^3
1	1	0.232627	0.124393
2	1	0.447806	0.236348
3	1	0.401281	0.335862
4	1	0.767668	0.422938
5	1	0.680433	0.559770
6	1	0.610645	0.497574

As we can see from Table 6 and Table 13, the BI and the FVI give the different order of influence of individual system components on the reliability of the human module in Fig. 2. Mainly, this is caused by the fact that the BI does not take the reliability of a component for which it is computed into account. The "*Incorrect interpretation of doctor's instructions*" has the most consequence on the human module reliability (it is defined by the BI) but this event is very rare and therefore, the probability that this event contribute to the system unreliability (it is defined by the FVI) is very small.

4 Discussion and Conclusions

The analysis on occurred medical mistake in the last decades has clearly shown that the organization and human factor play signification role in the risk of diagnosis and treatment [Bogner 1994, Dhillon 2003]. This is due also to the fact that the reliability of the technical components has significantly improved in recent years. As a consequence, the influence of the errors of the organizations managing and of the human operators to systems operation has significantly increased. Therefore correct reliability analysis of a healthcare system needs consideration of three basic component of the system [Zaitseva et al.2011]: technical (hardware and software), human and organization. The analysis of these components needs the similar algorithms, but in the present time the reliability estimation of these component are based on the different approaches and investigated by different methods. In this paper we develop one of possible way for reliability analysis of a healthcare system based on the interpretation of a healthcare system as MSS. New approach for the quantification of a health care system as MSS is developed. The background of this analysis is importance analysis that allows to estimate an influence of different system component functioning or failure to the system reliability (availability, performance) change.

In this paper new algorithms for calculation of IM (Table 2) for a MSS analysis are considered. These algorithms are implemented based on the mathematical approach of Multiple-Valued Logic as Direct Partial logic Derivative. Application of these measures in the analysis of healthcare system reliability is presented. In this paper these measures and algorithms for the calculation are used for the human module of the healthcare system that is not typical application of the methods of importance analysis. Proposed algorithms can be used for the other components of a healthcare system. In this case the investigated system includes technical components (hardware and software), human factor and organization component, and is interpreted as MSS. This interpretation of system allows investigating some performance levels of system functioning, that didn't include level of functioning and fault only. Therefore the proposed algorithms can be used for the investigation of a healthcare components (for example, as technical or human) and a reliability behaviour of a healthcare system at lardge.

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